Math 102

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  - Use relative behavior at 0 and ∞.
  - Even or odd?
  - Calculate zeroes if possible.

Just kidding!

Welcome to Math 102!

About myself

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- You can call me...

Good	Bad
Krishanu	Bro
Dr. Sankar	K-Dawg
Professor	Dude
etc.	etc.

- Contact: ksankar@math.ubc.ca

Lecture Policies

- Mathematics is not a spectator sport. Lecture will be interactive with occasional exercises. Bring something to write on. Working with your neighbors is encouraged!

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- **Note:** Lecture participation is not mandatory! I would rather you get some extra sleep, instead of playing games on your phone.

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- Limits and derivatives.
- Use to analyze functions.
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- You will also be required to **communicate ideas clearly and effectively.**

Math 102: Resources

- Course website:
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- Piazza (see link on Canvas)

Assignments and Grading

- WeBWorK (online, 3x/week) - 15%
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- Final exam (Date TBD) - 50% (You must score at least 44% on the final exam to pass the course)

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- Any regrade requests must be given in writing using the form on the course webpage.

Typical Math 102 Week

- Monday: Pre-lecture WeBWorK due
- Tuesday 9:30-11: Lecture
- Wednesday: Pre-lecture WeBWorK due
- Thursday 9:30-11: Lecture
- Thursday: WeBWork due
- (Every other Friday: OSH due)

Reminders

- All questions regarding registration or sectioning should be directed to Mark MacLean or Margaret Ness.
- Reminder: https://canvas.ubc.ca/courses/6219
- Don't panic. Within a week, this will feel less overwhelming.

Power Functions

- A function of the form $f(x) = ax^n$ (where a is a constant and n is a positive integer) is called a **power function**. For example,

Power Function	Not a Power Function
x^2	$\sqrt{x} = x^{1/2}$
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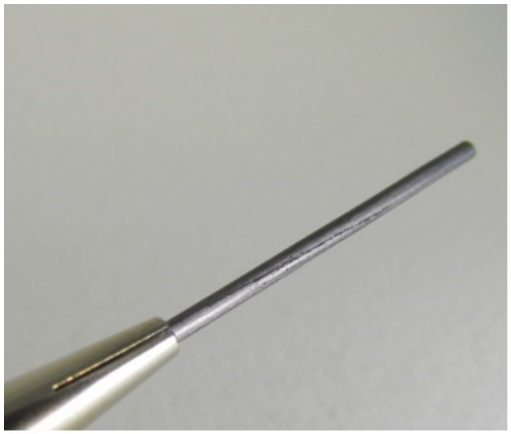
- Goals
  - Relative behavior as $x \to 0$ or $x \to \infty$
  - Calculating intersection points of power functions
  - Interpret results verbally, examples in nature

Example - Why are cells so small?

~100mm



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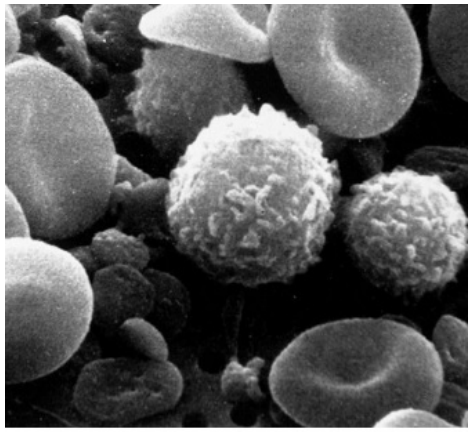

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https://en.wikipedia.org/wiki/White_blood_cell/media/File:SEM_blood_cells.jpg

- WBCs are 12-15 microns in diameter.

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- **Mathematical model:** assume the cell is *spherical*, and

  1. Absorption rate is proportional to surface area.
  2. Consumption rate is proportional to volume.

Example - Spherical Cells

$$A(r) = 4\pi k_1 r^2 \qquad C(r) = \frac{4}{3}\pi k_2 r^3$$

Question: Which of the following is true?

A. Absorption is greater than consumption for very large r and vice versa for small r.

B. Consumption is greater than absorption for very large r and vice versa for small r.

C. Both A and B are possible, depending on k_1 and k_2.

Asymptotic Behavior

- https://www.desmos.com/calculator/xrbtlbd8pk
- https://www.desmos.com/calculator/jzmjz1951u
- Example: Calculating the intersection of two power functions, in a specific case and also in general.

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In order for the cell to not starve, must have absorption $\geq$ consumption

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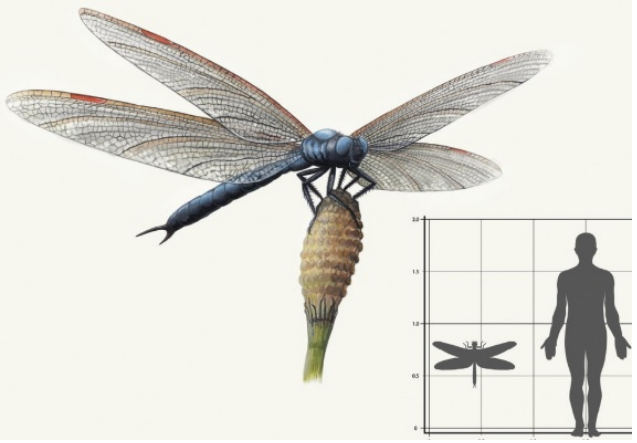
Therefore, $\boxed{r \leq \frac{3k_1}{k_2}}$.

Two other examples

Leg thickness relative to body size. Mass is proportional to volume, muscle strength is proportional to cross-sectional area.

Giant arthropods in the Carboniferous era - high atmospheric oxygen.



Graph Sketching

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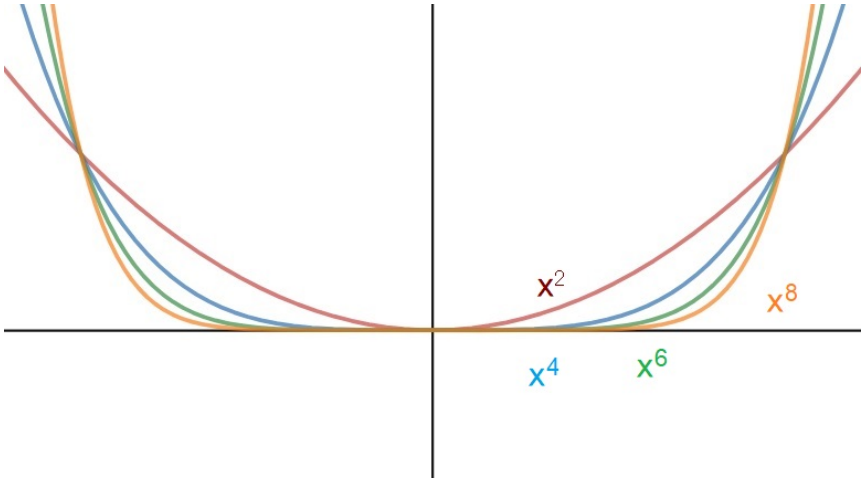
Goals:

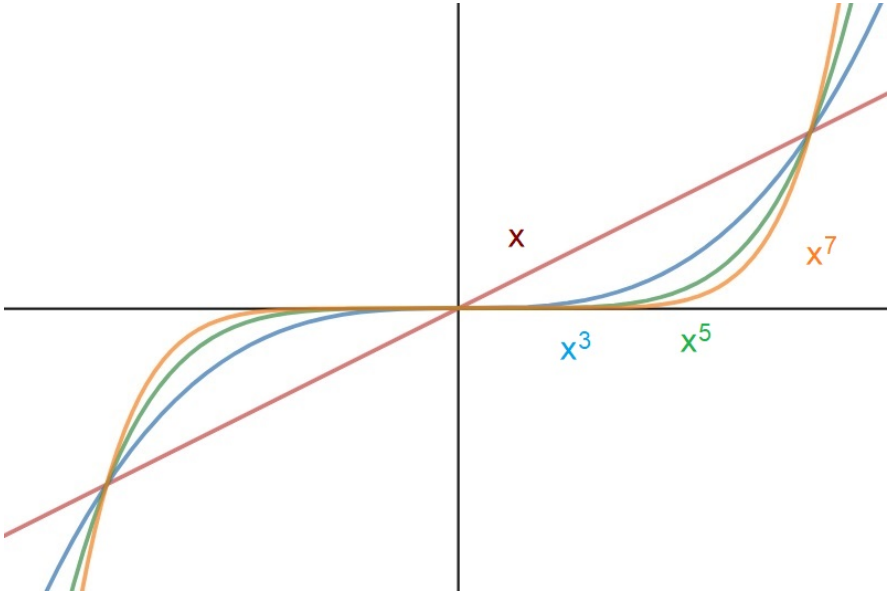
- Sketch power functions, identify even/odd functions.
- Learn to sketch graphs of simple polynomials such as $y = ax^m + bx^n$.
- Learn to sketch graphs of simple rational functions such as $y = \frac{ax^m + bx^n}{cx^k + dx^\ell}$.
- Gain intuition about how these simple functions behave.

Even and Odd Power Functions

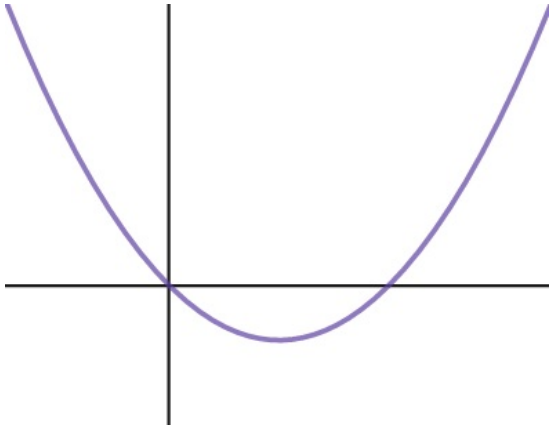
When degree is even: symmetry across the y-axis.

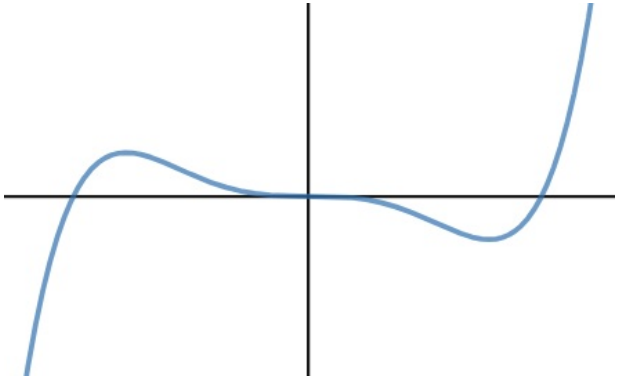


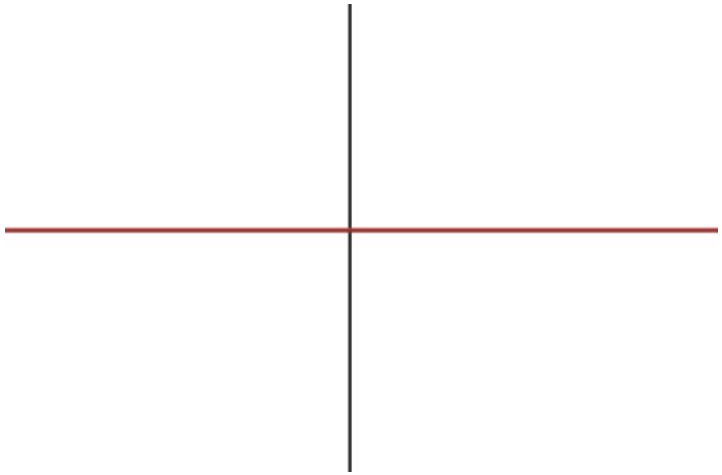
When degree is odd: symmetry through the origin.

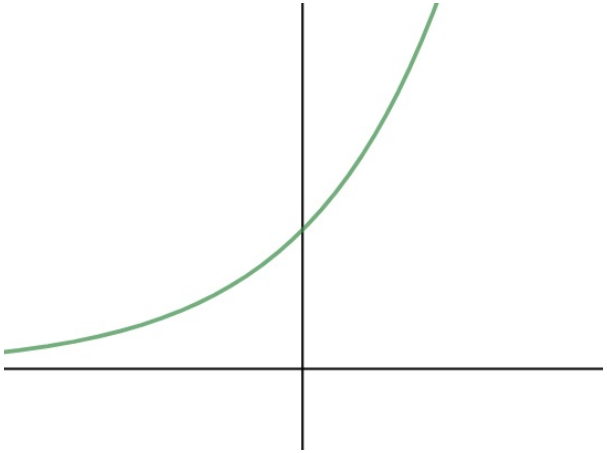


Even? Odd?









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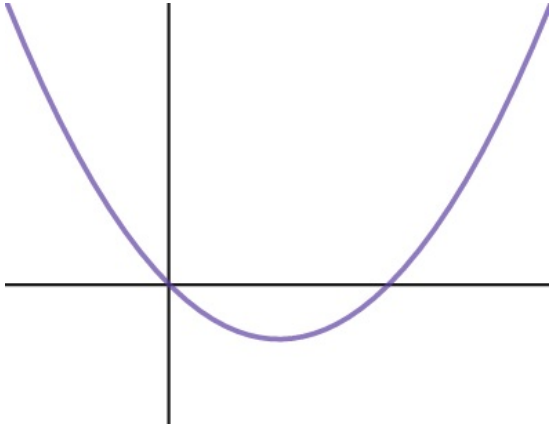
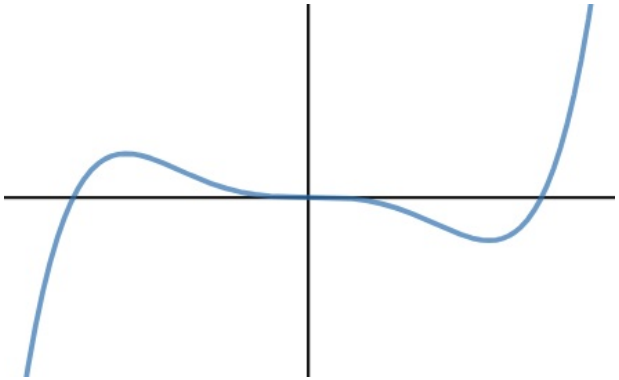
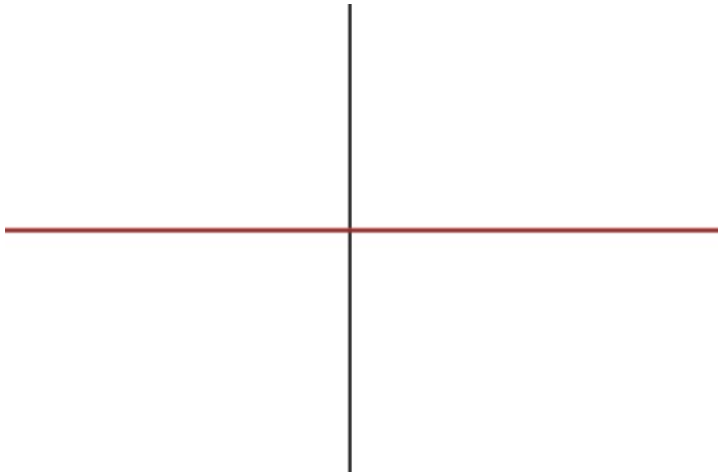


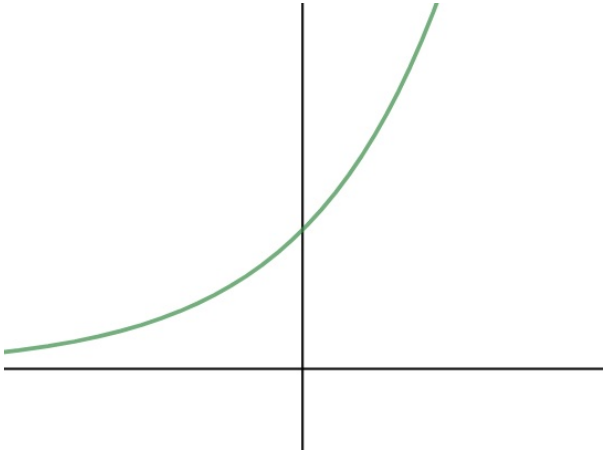
Neither! $y = x^2 - x$



Odd! $y = x^5 - x^3$



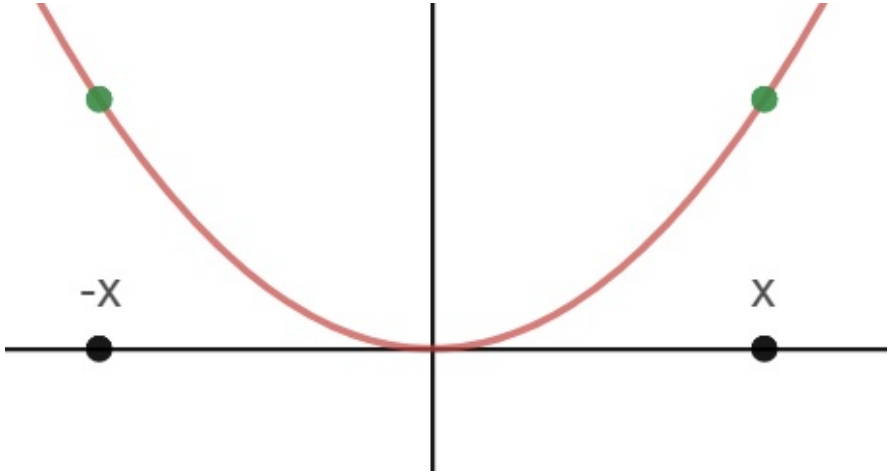
Both! $y = 0$



Neither! $y = e^x$

Even functions - Algebraically

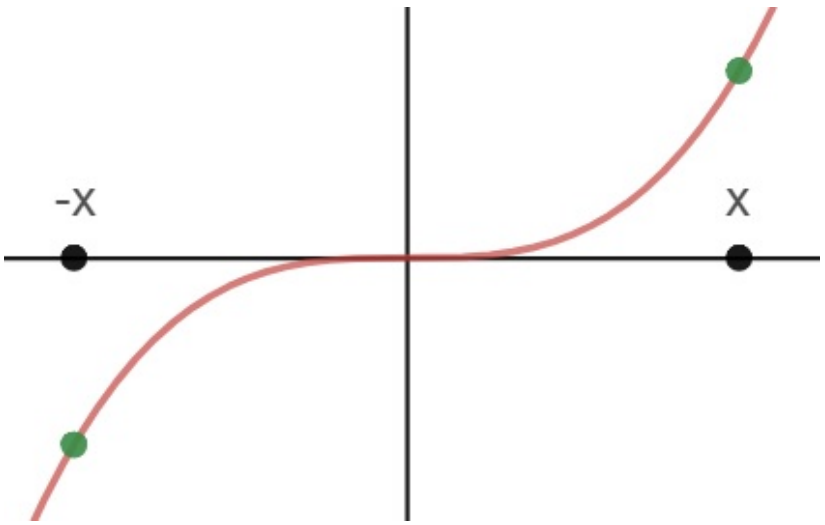
- A function $f(x)$ is **even** if $f(-x) = f(x)$ for all x.



- (To be done on board) Show that the function $f(x) = x^4 - x^2$ is even.

Odd functions - Algebraically

- A function $f(x)$ is **odd** if $f(-x) = -f(x)$ for all x.



- Exercise: Show that the function $f(x) = x^3 - \frac{1}{x}$ is odd.

Example: Sketching $y = x^3 - x$

To be done on board.

Exercise: Sketching $y = x^3 + x^2$

To be done as an exercise.

More general: Sketching $ax^3 + bx^2$

To be done on board if time available.

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- Your first assignments:
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